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SCALING BEHAVIOR OF BLACK HOLE ENTROPY

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Abstract

It is shown that the entropy of fourdimensional black holes in string theory compactified on weighted Calabi-Yau hypersurfaces shows scaling behavior in a certain limit. This leads to non-monotonic functions on the moduli space.

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1 Introduction

One of the longstanding concerns in string theory is the problem of the vacuum degeneracy, the hope being of identifying a dynamical principle which predicts the unique physical ground state of the heterotic string (or M & F-theory [1, 2, 3, 4]). A first step toward such a goal would be to find physically motivated quantities which are sensitive to the internal geometry and which define interesting functions on the moduli space.

Natural candidates for such functions are the Yukawa couplings and the free energy. Both of these quantities are functions over the $(h^{(1,1)} + h^{(2,1)})$ complex-dimensional moduli space of each individual Calabi-Yau manifold. These multidimensional components form the connected web of the collective moduli space [5] and therefore even if one could compute both quantities for a reasonable class of spaces one would have to compare sets of functions of different order. Future progress in technology should make this possible. To simplify this problem one can consider the large volume limit, neglecting the instanton contributions to these functions, and focus on the classical bulk part. In this limit both the Yukawa coupling and the free energy can be computed within the class of Calabi-Yau hypersurfaces embedded in weighted projective four-space [6]. It turns out that both quantities show scaling behavior with respect to a scaling variable defined by the dimension of the space of global functions of the hyperplane bundle on the manifold, and, as a consequence, satisfy a scaling relation between themselves [7]. The resulting critical exponents can be viewed as specific characteristics of this space of Calabi-Yau manifolds. Both functions considered in [7] turn out to be monotonic and therefore it is of interest to search for different types of characteristics of the moduli space.

Recently progress has been made in the understanding of the entropy of black holes in string theory [8]. In the present note it is pointed out that the entropy of N=2 black holes in type II compactifications on Calabi-Yau threefolds CY_3 leads to functions with minima on the collective moduli space.

2 Black Holes on Calabi-Yau Threefolds

N= 2 Calabi-Yau black holes embedded in type IIA string theory can also be viewed as solutions of M-theory compactified on $\mathrm{CY}_3 \times \mathrm{S}^1$. In these theories there are $(h^{(1,1)} + 1)$ electric charges (q_0, q_A) associated to D0-branes and the D2-branes wrapped around the $h^{(1,1)}$ 2-cycles in the threefold. Dual to these are the magnetic charges (p^0, p^A) of the D6-brane wrapped around the threefold and the D4-branes wrapped around the 4-cycles. In the large volume limit the entropy of black holes which carry only 0-brane and 4-brane charges (q_0, p^A) the entropy has been found in [9, 10, 11, 12, 13] to be determined by the central charge of the theory by solving the Ferrara–Kallosh equation [11]. The general solution of this extremization condition for the central charge has not been found yet but for a number of types of black holes this relation has been solved. Useful in the present context are axion–free black holes for which the entropy is determined by the classical Yukawa couplings $C_{ABC} = \int_{\mathrm{CY}_3} \omega_A \omega_B \omega_C$, where $\omega_\Lambda \in \mathrm{H}^2(\mathrm{CY}_3)$, and the linear form of the first Pontrjagin class. Interesting probes of the moduli space can be provided by black holes with nonvanishing charges (q_0, p^A). For such black holes the entropy is given by [12]

$$S \sim \frac{2\pi}{\sqrt{6}} \sqrt{q_0 C_{ABC} p^A p^B p^C} \tag{1}$$

for large charges. The microscopic derivation of this entropy as a gas of D0-branes obtained from intersecting D4-branes has been described in refs. [14, 15].

As will become clear in the following section, a more interesting function on the moduli space can be obtained by taking into account a shift in the theta angle. It was shown in [12] that such a shift leads to an entropy of black holes with D0-brane and D4-brane charges in the large volume limit which is of the form

$$S \sim \frac{\pi}{6} \sqrt{(24 q_0 + c_2 J_A p^A)\, C_{ABC} p^A p^B p^C}. \tag{2}$$

The microscopic origin of this shift is to be found in the additional D0-brane contribution coming from the anomaly in type IIA string theory [16, 17] as pointed out in [15].

For fixed charges these are complicated functions on each of the components of the collective moduli space which will have a rather involved, perhaps unilluminating, behavior.

3 Scaling behavior of Black Hole Entropy

In order to obtain simpler probes of the moduli space it is more useful to focus on the bulk contribution to the entropy which originates from the universal deformation defined by the restriction of the hyperplane bundle of the ambient space. Denoting $q_0 \equiv q$ and $p^1 \equiv p$ and $C_{111} \equiv C$ we focus on black holes with electric-magnetic charges (p, q) such that the entropy simplifies to

$$S_{p,q} \sim 2\pi\sqrt{qCp}. \tag{3}$$

Considering anti-D0-branes of charge $q_0 = -q$ the anomaly shifted entropy in turn becomes

$$S^{\theta}_{p,q} \sim \frac{\pi p^2}{6}\sqrt{\left(-24\frac{q}{p} + c_2 \cdot L\right) C}. \tag{4}$$

The Yukawa couplings of the universal Kähler deformation induced by the ambient Kähler form is given by the degree the degree of a natural line bundle which on hypersurfaces embedded in weighted projective space is induced by the hyperplane bundle of the ambient space. This leads to

$$C \equiv \int_M c_1^3(L). \tag{5}$$

These couplings were computed in [7] for the class of weighted Calabi-Yau hypersurface threefolds constructed in [6]. For such spaces the natural candidate for a line bundle is the pullback of the weighted form of the hyperplane bundle

$$L = \mathcal{O}^{(k)}_{\mathbf{P}_{(k_1,\ldots,k_5)}} \tag{6}$$

on the weighted ambient space [18, 19]

$$k = lcm\{\ \{gcd(k_i, k_j)|\ i, j = 1, .., 5; i \neq j\} \cup \{k_i | k_i \text{ does not divide } \sum_{i=1}^{5} k_i\}\}. \tag{7}$$

The pullback $j^*(\mathcal{O}^{(k)}_{\mathbf{P}_{(k_1,\ldots,k_5)}})$ of L from the ambient space to the embedded Calabi–Yau manifold $j : M \longrightarrow \mathbb{P}_{(k_1,\ldots,k_5)}$ induces an antigeneration $j^*(c_1(L))$, which will also be denoted by L. The Yukawa coupling $C(L)$ of this antigeneration leads, in the large radius limit, to the expression

$$C = \int_M (j^*(c_1(\mathcal{O}^{(k)}_{\mathbf{P}_{(k_1,\ldots,k_5)}}))^3 = \left(\frac{\sum_{i=1}^5 k_i}{\prod_{i=1}^5 k_i}\right)\ k^3. \tag{8}$$

The result shows that for this class this Yukawa coupling defines a scaling type function on the collective moduli space which is of the form

$$C \sim \frac{6h-16}{1+ah^{-\alpha}},$$

where $h = \dim \mathrm{H}^0(\mathrm{CY}_3, L)$ is the dimension of the space of global functions on the individual components of the large moduli space. This dimension can be computed for the bundles $\mathcal{O}^{(k)}_{\mathbf{P}_{(k_1,\ldots,k_5)}}$ as

$$h^0(\mathcal{O}^{(k)}_{\mathbf{P}_{(k_1,\ldots,k_5)}}) = \frac{1}{k!}\frac{\partial^k}{\partial t^k}\left(\frac{\left(1-t^{\sum_i k_i}\right)}{\prod_i\left(1-t^{k_i}\right)}\right)\Bigg|_{t=0}. \tag{9}$$

The constants appearing in this scaling relation are approximately $(a,\alpha) = (5, 0.7)$. This leads for singly charged black holes to an entropy at zero theta angle which scales for large couplings as

$$S_{p,q} \sim s_{p,q}\, h^{1/2}, \tag{10}$$

where $s_{p,q}$ is constant.

The entropy at nonvanishing theta angle again leads to a more interesting scaling form by using the second scaling relation described in [7] for the linear form defined by the second Chern class on the second cohomology group

$$c_2 \cdot L \equiv \int_M c_1(L) \wedge c_2(M) \tag{11}$$

where $c_2(M)$ is the second Chern class of the Calabi–Yau 3–fold. This number has been of relevance in [20] where it was shown that the generalized index introduced in [21]

$$\mathcal{F} = \frac{1}{2}\int \frac{d^2\tau}{\tau_2}\; Tr\left[(-1)^F F_L F_R q^{H_L}\bar{q}^{H_R}\right]. \tag{12}$$

describes the one–loop partition function of the twisted N=2 theory coupled to gravity. Here the integral is over the fundamental domain of the moduli space of the torus, $F_{L,R}$ denote the left and right fermion numbers and the trace is over the Ramond sector for both the left– and right–movers [1]. It was shown in [20] that this generalized index reduces in lowest order to

$$\mathcal{F}^{\uparrow} = \frac{1}{24}\int_M K \wedge c_2(M) \tag{13}$$

[1]The contribution of the ground states of the supersymmetric Ramond sector to $\mathcal{F}$ has to be deleted in order for the integral to converge.

where K is the Kähler form of the manifold. Thus the numbers (11) define the universal contribution $\mathcal{F}_L^{\uparrow} = L \cdot c_2/24$ to the large radius limit of this partition function.

The resulting scaling relation now takes the form

$$c_2 \cdot L \sim b\, C^{\beta}, \tag{14}$$

where $(b, \beta) = (36, 0.3)$. Thus the entropy for large coupling scales like

$$S^{\theta}_{p,q}(C) \sim \frac{\pi^2 p^4}{36} \sqrt{\left(-\frac{24q}{p} + bC^{\beta}\right) C}. \tag{15}$$

For $q/p << C$ this leads to the critical exponent $\gamma_C = \frac{\beta+1}{2}$.

More interestingly, the shifted black hole entropy defines a probe which leads to nonmonotonic functions on the moduli space. Considering for convenience the square of the entropy one finds finds that for fixed charges $S^2_{(1,1)}(C)$ takes its minimum value at

$$C_{min} = \left(\frac{q}{p} \frac{24}{b(\beta+1)}\right)^{\frac{1}{\beta}}. \tag{16}$$

The behavior of the function $S^{\theta}_{p,q}(C)$ is illustrated in Figure 1.

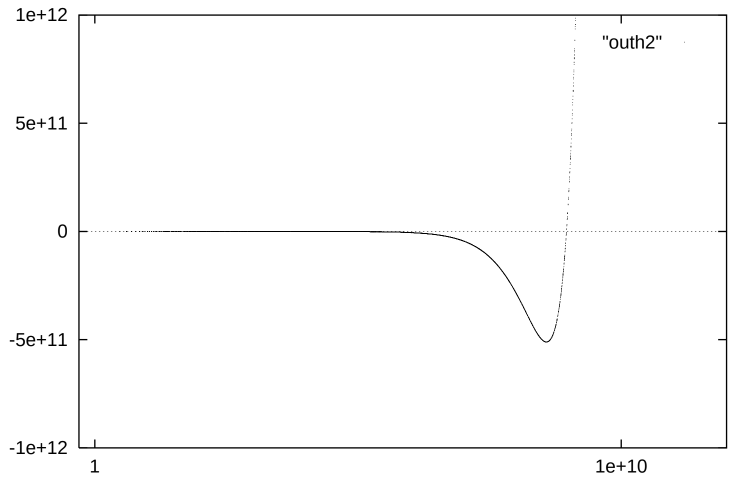

Figure 1: $(S^{\theta}_{q,p})^2$ *at* $q/p = 10^3$ *for the class of hypersurface threefolds [6].*

Combining the two scaling relations 2 and 14 leads to the following behavior of the square of the entropy as a function of the scaling variable h

$$(S^\theta_{p,q})^2(h) \sim \frac{\pi^2 p^4}{6}\left[-\frac{q}{p}+\frac{b}{24}\left(\frac{6h-16}{1+ah^{-\alpha}}\right)^\beta\right]\frac{6h-16}{1+ah^{-\alpha}}, \tag{17}$$

with asymptotic exponent $\gamma_h = 1+\beta$.

In summary the entropy of black holes in string theory leads to the first non-monotonic probes of the collective moduli spaces of Calabi-Yau vacua.

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References

[1] P. Townsend, Phys. Lett. **B350**(1995)184, hep-th/9501068; J. Schwarz, Phys. Lett. **B367** (1996) 97, hep–th/9510086; E. Witten, Nucl. Phys. **B443**(1995)85, hep-th/9503124

[2] P. Horava and E. Witten, Nucl. Phys. **B460** (1996) 506, hep–th/9510209; Nucl. Phys. **B475**(1996)94, hep–th/9603142

[3] C. Vafa, Nucl. Phys. **B469** (1996) 403, hep–th/9602022

[4] D. R. Morrison and C. Vafa, Nucl. Phys. **B473**(1996)74, hep–th/9602114; *ibid* **B476**(1996)437, hep–th/9603161

[5] P. Candelas, C.A. Lütken, A. Dale and R. Schimmrigk, Nucl.Phys. **B298**(1988)493
P. Candelas, P.Green and T.Hübsch, Nucl. Phys. **B330**(1990)
M. Lynker and R.Schimmrigk, Nucl.Phys. **B484**(1997)562, hep-th/9511058

T. Chiang, B. Greene, M. Gross and Y. Kantor, Nuc. Phys. Proc. Suppl. **46**(1996)82, hep-th/9511204

A. Avram, P. Candelas, D. Jancic and M. Mandelberg, Nucl. Phys. **B465**(1996)458; hep-th/9511230;

A. Avram, M. Kreuzer, M. Mandelberg and H. Skarke, Nucl. Phys. **B505**(1997)625, hep-th/9703003

[6] P. Candelas, M. Lynker and R. Schimmrigk, Nucl.Phys. **B341**(1990)383
A. Klemm and R. Schimmrigk, Nucl.Phys. **B411**(1994)559, hep-th/9204060
M. Kreuzer and H. Skarke, Nucl.Phys. **B338**(1992), hep-th/9205004

[7] R.Schimmrigk, Phys.Lett. **B388**(1996)60, hep-th/9412077; Mirror Symmetry II, alg-geom/9612012

[8] A. Strominger and C. Vafa, Phys.Lett. **B379**(1996)99, hep-th/9601029

[9] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. **D52**(1996)5412, hep-th/9508072

[10] A. Strominger, Phys. Lett. **B382**(1996)39, hep-th/9602111

[11] S. Ferrara and R. Kallosh, Phys. Rev. **D54**(1996)1514, hep-th/9602139; Phys. Rev. **D54**(1996)1525, hep-th/9603090

[12] K. Behrndt, G. L. Cardoso, B. de Wit, R. Kallosh, D. Lüst and T. Mohaupt, Nucl. Phys. **B488**(1997)236, hep-th/9610105

[13] S. J. Rey, Nucl. Phys. bf B508(1997)569, hep-th/9610157

[14] K. Behrndt and T. Mohaupt, Phys. Rev. **D56**(1997)2206, hep-th/9611140

[15] J. Maldacena, Phys. Lett. **B403**(1997)20, hep-th/9611163

[16] C. Vafa and E. Witten, Nucl. Phys. **B447** (1995) 85, hep-th/9505053

[17] M. Bershadsky, V. Sadov and C. Vafa, Nucl. Phys. **B463** (1996) 420, hep-th/9511222

[18] P.Berglund and S.Katz, Nucl. Phys. **B420**(1994)289, hep-th/9311014

[19] D.Dais, unpublished

[20] M. Bershadsky, S. Cecotti, H. Ooguri, and C. Vafa, Nucl.Phys. **B405**(1993)279, hep-th/9302103; Comm. Math. Phys. **165**(1994)311, hep-th/9309140

[21] S.Cecotti, P.Fendley, K.Intriligator and C.Vafa, Nucl.Phys. **B386**(1992)405, hep-th/9204102;
I. Antoniadis, E. Gava and K.S. Narain, Nucl. Phys. **B383**(1992)93, hep-th/9204030